Fall 2025

MATH 007A: Calculus for Life Sciences I

Midterm 1 Review

Solutions

Problem 1. Evaluate the following limits:

(a) $\lim_{x\to 2} \dfrac{x^2 - x - 2}{x^2 - 2x}$

Solution. First, factor the numerator and denominator:

$$\frac{x^2 - x - 2}{x^2 - 2x} = \frac{(x+1)(x-2)}{x(x-2)}.$$

Cancel the common factor $(x-2)$ (valid for $x \neq 2$):

$$\frac{(x+1)(x-2)}{x(x-2)} = \frac{x+1}{x}.$$

Now take the limit:

$$\lim_{x\to 2} \frac{x+1}{x} = \frac{2+1}{2} = \frac{3}{2}.$$

(b) $\lim_{x\to 0} \dfrac{\sin(4x)}{5x}$

Solution. Multiply and divide by 4 to make the expression resemble the standard limit $\lim_{u\to 0} \frac{\sin(u)}{u} = 1$:

$$\frac{\sin(4x)}{5x} = \frac{\sin(4x)}{4x} \cdot \frac{4}{5}.$$

Then take the limit:

$$\lim_{x\to 0} \frac{\sin(4x)}{4x} = 1 \quad \text{so} \quad \lim_{x\to 0} \frac{\sin(4x)}{5x} = \frac{4}{5}.$$

(c) $\lim_{x\to -\infty} \dfrac{x^7 - 8x^6 + 4}{5x^4 + 2x^2 - 1}$

Solution. For large negative x, the highest-degree terms dominate:

$$\frac{x^7 - 8x^6 + 4}{5x^4 + 2x^2 - 1} \approx \frac{x^7}{5x^4} = \frac{1}{5}x^3.$$

Since $x^3 \to -\infty$ as $x \to -\infty$ and $\frac{1}{5} > 0$, the limit is

$$\lim_{x\to -\infty} \frac{x^7 - 8x^6 + 4}{5x^4 + 2x^2 - 1} = -\infty.$$

Problem 2. Consider the function

$$f(x) = \begin{cases} 1 + bx^2 & \text{if } x < 1, \\ a\sqrt{x} & \text{if } x \geq 1. \end{cases}$$

(a) Compute $\lim_{x\to 1^-} f(x)$ in terms of b.

Solution. As $x \to 1^-$, we use the first case:

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (1 + bx^2) = 1 + b \cdot 1^2 = 1 + b.$$

(b) Compute $\lim_{x \to 1^+} f(x)$ in terms of a.

Solution. As $x \to 1^+$, we use the second case:

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} a\sqrt{x} = a.$$

(c) Use your answers in parts (a) and (b) to find a condition on a and b such that $f(x)$ is continuous at $x = 1$.

Solution. For continuity at $x = 1$, the left and right limits must be equal:

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) \quad \Rightarrow \quad 1 + b = a.$$

(d) Compute the left-hand derivative of $f(x)$ at $x = 1$, i.e. $\lim_{x \to 1^-} f'(x)$.

Solution. For $x < 1$, $f(x) = 1 + bx^2$, so

$$f'(x) = 2bx \quad \Rightarrow \quad \lim_{x \to 1^-} f'(x) = 2b.$$

(e) Compute the right-hand derivative of $f(x)$ at $x = 1$, i.e. $\lim_{x \to 1^+} f'(x)$.

Solution. For $x \geq 1$, $f(x) = a\sqrt{x} = ax^{1/2}$, so

$$f'(x) = \frac{a}{2\sqrt{x}} \quad \Rightarrow \quad \lim_{x \to 1^+} f'(x) = \frac{a}{2}.$$

(f) Use your answers in parts (c), (d), and (e) to find a system of equations that ensures $f(x)$ is both continuous and differentiable at $x = 1$.

Solution. For continuity: $a = 1 + b$. For differentiability: $2b = \frac{a}{2}$. So the system is:

$$\begin{cases} a = 1 + b, \\ 2b = \frac{a}{2}. \end{cases}$$

(g) Solve the system to find the values of a and b such that $f(x)$ is continuous and differentiable at $x = 1$.

Solution. Substitute $a = 1 + b$ into the second equation:

$$2b = \frac{1 + b}{2} \quad \Rightarrow \quad 4b = 1 + b \quad \Rightarrow \quad 3b = 1 \quad \Rightarrow \quad b = \frac{1}{3}.$$

Then $a = 1 + \frac{1}{3} = \frac{4}{3}$.

Problem 3. Let $f(x) = -\frac{1}{2x}$.

(a) Use the limit definition of the derivative to compute $f'(x)$.

Solution. Using the definition:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{-\frac{1}{2(x+h)} + \frac{1}{2x}}{h}.$$

Combine the fractions in the numerator:

$$\lim_{h \to 0} \frac{1}{h} \cdot \left(\frac{1}{2x} - \frac{1}{2(x+h)} \right) = \lim_{h \to 0} \frac{1}{h} \cdot \frac{(x+h) - x}{2x(x+h)} = \lim_{h \to 0} \frac{1}{2x(x+h)}.$$

Taking the limit as $h \to 0$, we get:

$$f'(x) = \frac{1}{2x^2}.$$

(b) Find the equation of the tangent line at $\left(-2, \frac{1}{4}\right)$.

Solution. From part (a), $f'(x) = \frac{1}{2x^2}$, so

$$f'(-2) = \frac{1}{2 \cdot (-2)^2} = \frac{1}{2 \cdot 4} = \frac{1}{8}.$$

The equation of the tangent line is

$$y - \frac{1}{4} = \frac{1}{8}(x+2) \Leftrightarrow y = \frac{1}{8} \cdot x + \frac{1}{8} \cdot 2 + \frac{1}{4} \Leftrightarrow y = \frac{1}{8} \cdot x + \frac{1}{2}.$$

Problem 4. Let $f(x) = 2x^2(x - 4x^3)$.

(a) Use the product rule to compute $f'(x)$.

Solution. Let $u = 2x^2$ and $v = x - 4x^3$. Then,

$$f'(x) = u'v + uv' = 4x(x - 4x^3) + 2x^2(1 - 12x^2).$$

Expand both terms:

$$4x(x - 4x^3) + 2x^2(1 - 12x^2) = 4x^2 - 16x^4 + 2x^2 - 24x^4 = 6x^2 - 40x^4.$$

(b) Expand $f(x)$.

Solution.

$$f(x) = 2x^2(x - 4x^3) = 2x^3 - 8x^5.$$

(c) Use the power rule to differentiate the expanded expression.

Solution. Differentiate term by term:

$$f'(x) = \frac{d}{dx}(2x^3 - 8x^5) = 6x^2 - 40x^4.$$

(d) Do both methods agree?

Solution. Yes, the derivative computed via the product rule and the power rule both give $f'(x) = 6x^2 - 40x^4$.

Problem 5. Suppose $N(t) = 10 + \frac{2\sin(t)}{e^{0.3t}}$ models a population (in millions) at time t (in years). What is the limiting population as $t \to \infty$?

Solution. As $t \to \infty$, the $e^{0.3t}$ term goes to ∞, while $\sin(t)$ remains bounded between -1 and 1. Therefore,

$$\lim_{t \to \infty} \frac{2\sin(t)}{e^{0.3t}} = 0.$$

Thus, the limiting population is $\lim\limits_{t \to \infty} N(t) = 10 + 0 = 10$ million people.