Fall 2025

MATH 007A: Calculus for Life Sciences I

Midterm 1 Review

Problem 1. Evaluate the following limits:

(a) $\lim_{x \to 2} \dfrac{x^2 - x - 2}{x^2 - 2x}$

(b) $\lim_{x \to 0} \dfrac{\sin(4x)}{5x}$

(c) $\lim_{x \to -\infty} \dfrac{x^7 - 8x^6 + 4}{5x^4 + 2x^2 - 1}$

Consider the function

$$f(x) = \begin{cases} 1 + bx^2 & \text{if } x < 1, \\ a\sqrt{x} & \text{if } x \geq 1. \end{cases}$$

(a) Compute $\lim_{x \to 1^-} f(x)$ in terms of b.

(*b*) Compute $\lim_{x \to 1^+} f(x)$ in terms of a.

(*c*) Use your answers in parts (a) and (b) to find a condition on a and b such that $f(x)$ is continuous at $x = 1$.

(*d*) Compute the left-hand derivative of $f(x)$ at $x = 1$, i.e. $\lim_{x \to 1^-} f'(x)$.

(*e*) Compute the right-hand derivative of $f(x)$ at $x = 1$, i.e. $\lim_{x \to 1^+} f'(x)$.

(*f*) Use your answers in parts (c), (d), and (e) to find a system of equations that ensures $f(x)$ is both continuous and differentiable at $x = 1$.

(*g*) Solve the system to find the values of a and b such that $f(x)$ is continuous and differentiable at $x = 1$.

Problem 3. Let $f(x) = -\frac{1}{2x}$.

(*a*) Use the limit definition of the derivative to compute $f'(x)$.

(*b*) Find the equation of the tangent line at $\left(-2, \frac{1}{4}\right)$.

Problem 4. Let $f(x) = 2x^2(x - 4x^3)$.

(*a*) Use the product rule to compute $f'(x)$.

(*b*) Expand $f(x)$.

(*c*) Use the power rule to differentiate the expanded expression.

(*d*) Do both methods agree?

Problem 5. Suppose $N(t) = 10 + \frac{2\sin(t)}{e^{0.3t}}$ models a population (in millions) at time t (in years). What is the limiting population as $t \to \infty$?