Summer 2025

MATH 149A: Probability and Mathematical Statistics

Midterm Review Problems

Probability distributions on finite sets

1. You have a bag with 4 red marbles and 3 blue marbles. What is the probability of drawing a red marble?
2. If you flip a fair coin 5 times, what is the probability of getting exactly 2 heads?
3. If you flip a fair coin 10 times, what is the probability of getting at least 2 heads?
4. Among 35 students in a class, 17 earned 'A' on the midterm, 14 earned 'A' on the final exam, and 11 did not earn 'A' on either exam. What is the probability that a randomly selected student from this class earned 'A' on both exams?

Counting principles

1. How many different words can be produced from the letters of the word '`MISSISSIPPI`'?
2. In a department consisting of 10 students and 8 faculty members, a committee is to be formed. The committee will consist of a president and a vice president, both of whom must be chosen from the faculty members. Additionally, 3 students will be chosen as members of the committee. How many different committees can be formed?
3. A password consists of 4 letters (A-Z) followed by 3 digits (0-9). How many different passwords are possible?

Conditional probability and Bayes' formula

1. A pair of fair six-sided dice is rolled once.
   (*a*) Find the probability that there is at least one six.
   (*b*) Find the probability that both dice show sixes, given that there is at least one six.
2. A factory produces two types of widgets: Type A and Type B. Type A accounts for 10% of total production, while the remaining 90% are Type B. Of the Type A widgets, 95% pass quality control, while 90% of Type B widgets pass.
   If a randomly selected widget has passed quality control, what is the probability that it is of Type A?
3. An airport security system correctly detects 99% of explosive materials, but it also gives false alarms for 2% of non-explosive items. Out of all checked items, 5% contain explosives. If a randomly selected item sets off the alarm, what is the probability that it actually contains explosives?

Independence

1. Consider two events A and B such that $P(A) = 0.3$ and $P(B) = 0.4$. If A and B are independent, what is $P(A \cap B)$?
2. Consider two events E and F such that $P(E) = 0.6$ and $P(F) = 0.7$. If E and F are independent, what is $P(E \cup F)$?

3. In a survey, it was revealed that 75% of people like spicy food, and 20% of people are vegetarians. Among those who like spicy food, 10% are vegetarians. Are the events "liking spicy food" and "being a vegetarian" independent?

4. You have a deck of 52 cards. Events J and K are defined as follows:

$$J = \text{Drawing a red card}$$
$$K = \text{Drawing a face card}$$

Are these events independent?

Random variables, PMF, PDF, CDF and expectation

1. Let X be a random variable representing the number of heads obtained when flipping a coin 3 times. Find the probability mass function of X.

2. Let Y be the random variable representing the number of times a 6 appears when a fair six-sided die is rolled 4 times.

   (*a*) Find the PMF of Y.

   (*b*) Find the expected value of Y.

3. Suppose that the sample space S contains four elements $\{-1, 1, 2, 3\}$, with probabilities 0.1, 0.4, 0.2, and 0.3 respectively. Suppose $X(s) = s^2 - 4$ for $s \in S$. Compute the expected value $\mathbf{E}(X)$.

4. Let X be a random variable whose PDF is given by

$$f(x) = \begin{cases} c & \text{if } -3 \leq x \leq 1, \\ 0 & \text{else.} \end{cases}$$

   (*a*) Determine the value of c.

   (*b*) Compute the expected value $\mathbb{E}(X)$.

   (*c*) Let $Y = 5 - \frac{X}{2}$ and find the CDF of Y.