MATH 149A: Probability and Mathematical Statistics

Homework 1

Problem 1. (20 points) Let C_n be the half-open interval $I_n = \left(\frac{2}{n+1}, \frac{2}{n}\right]$ for each positive integer n.

(a) (6 points) Sketch the first three intervals I_1, I_2 and I_3, find the union $I_1 \cup I_2 \cup I_3$ and intersection $I_1 \cap I_2 \cap I_3$.

(b) (7 points) Find the union $\bigcup_{n=1}^{\infty} I_n$.

(c) (7 points) Find the intersection $\bigcap_{n=1}^{\infty} I_n$.

Problem 2. (15 points) A survey was conducted among 150 people regarding their coffee preferences. It was found that 90 people prefer black coffee, 75 people prefer lattes, and 30 people like both black coffee and lattes.

(a) (10 points) Let B be the set of people who prefer black coffee, and L be the set of people who prefer lattes. Express the number of people who do not prefer either type of coffee in terms of $|B|$, $|L|$, $|B \cap L|$, and the total number of people $|U|$.

(b) (5 points) Find how many people do not prefer either type of coffee.

Problem 3. (20 points) An urn contains 5 red balls, 4 green balls, and 3 blue balls. If you draw 4 balls at random without replacement, what is the probability of getting

(a) (10 points) exactly 2 green balls?

(b) (10 points) at least 2 green balls?

Problem 4. (10 points) Consider the word '`success`'.

(a) (5 points) How many possible arrangements of the letters in this word are there?

(b) (5 points) How many of these arrangements are different?

Problem 5. (15 points) A standard deck of 52 cards contains 4 suits ($\{\heartsuit, \diamondsuit, \spadesuit, \clubsuit\}$) of 13 cards each. What is the probability of drawing 2 $\heartsuit$ and 1 $\spadesuit$ in 3 draws without replacement?

Problem 6. (10 points) A club has 12 members. In how many ways can a president, vice-president, and secretary be chosen?

Problem 7. (10 points) If you roll two fair six-sided dice, what is the probability of getting

(a) (5 points) a sum between 7 and 9 (inclusive)?

(b) (5 points) a sum strictly less than 7 or strictly greater than 9?