MATH 146B: Ordinary and Partial Differential Equations

Homework 2

Boundary Value Problems

Problem 1. Find all solutions to the provided boundary value problem, or demonstrate that no solution exists.

(a) (5 points) $y'' + 25y = 0, y(0) = -2, y\left(\frac{\pi}{10}\right) = 7$.

(b) (5 points) $x^2y'' - 3xy' + 3y = 0, y(1) = 5$ and $y(-1) = -5$.

Eigenfunctions

Problem 2. Consider the differential operator $\mathcal{D}$. For each real number $\lambda \in \mathbb{R}$, find a basis for the space of continuous eigenfunctions of $\mathcal{D}$ with eigenvalue λ.

(a) (5 points) $\mathcal{D} = 2 \cdot \frac{d}{dx}$

(b) (5 points) $\mathcal{D} = \frac{d}{dx} + 2$

(c) (5 points) $\mathcal{D} = \frac{d^3}{dx^3}, \lambda = 0$