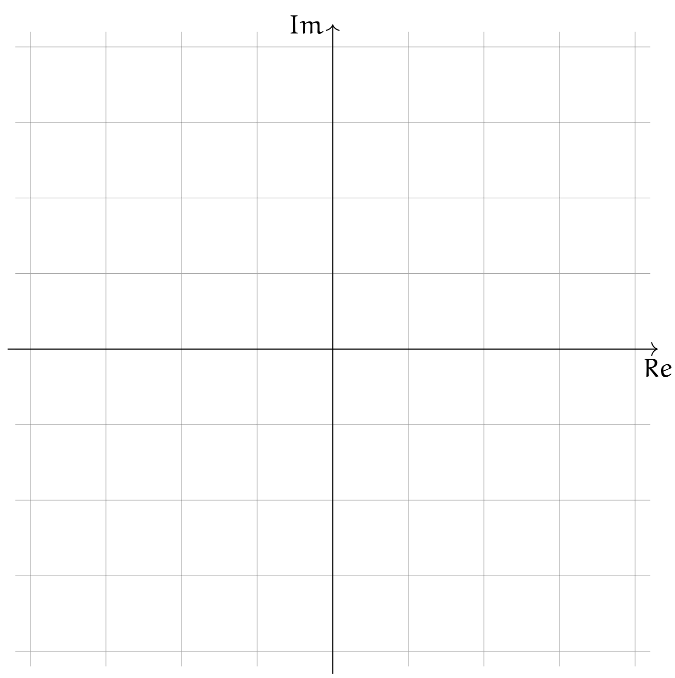
MATH 146B: Ordinary and Partial Differential Equations

Homework 1

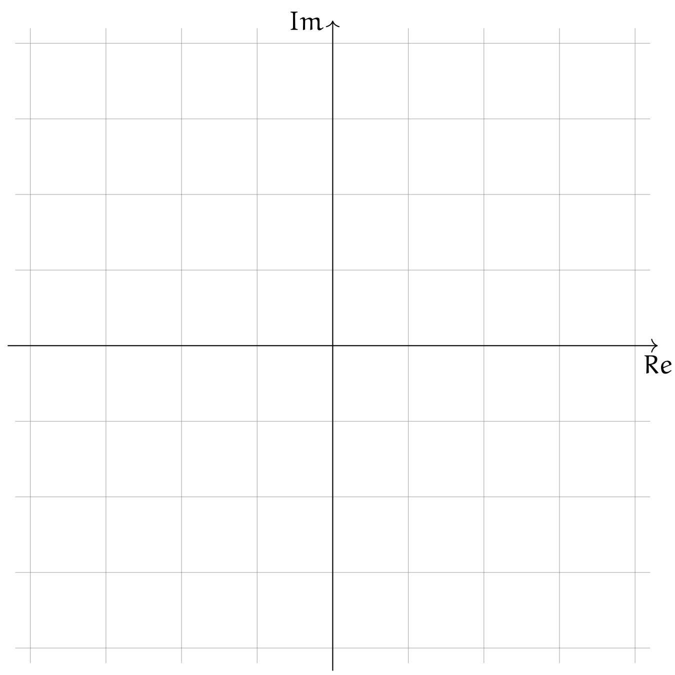
Roots of polynomials and solutions of ODE

Problem 1. Find the roots of the polynomial and sketch them on the complex plane.

(a) (10 points) $r^6 + 3^6 = 0$.



(b) (10 points) $r^5 - 32i = 0$.



Problem 2. (10 points) Given that $r^9 - 15r^8 + 75r^7 - 125r^6 + 729r^3 - 10935r^2 + 54675r - 91125 = \left(r^6 + 3^6\right)(r-5)^3$, use your answer in Problem $1(a)$ to find the general solution of the differential equation

$$y^{(9)} - 15y^{(8)} + 75y^{(7)} - 125r^{(6)} + 729y''' - 10935y'' + 54675y' - 91125y = 0.$$

Problem 3. (10 points) Find the general solution of the differential equation $y^{(6)} + y'' = 0$.

The method of undetermined coefficients

Problem 4. Use the method of undetermined coefficients to find the general solution of the differential equation $y^{(4)} - y = 3x + \cos(-3x) - 7e^{2x}$.

(a) (4 points) Solve the complementary equation $y^{(4)} - y = 0$.

(b) (12 points) Find particular solutions y_{p_1}, y_{p_2} and y_{p_3} of equations $y^{(4)} - y = 3x$, $y^{(4)} - y = \cos(-3x)$ and $y^{(4)} - y = -7e^{2x}$.

(c) (4 points) Use your answers in (b) and principle of superposition to find a particular solutions y_p of equation $y^{(4)} - y = 3x + \cos(-3x) - 7e^{2x}$ and write down the general solution of the initial equation.

The method of variation of parameters

Problem 5. (10 points) Consider the second-order differential equation $y'' + p(x)y' + q(x)y = F(x)$. Let y_1 and y_2 be solutions to the complementary homogeneous equation $y'' + p(x)y' + q(x)y = 0$. Show that for $y = u_1y_1 + u_2y_2$ with additional condition $u_1'y_1 + u_2'y_2 = 0$, the initial equation $\mathcal{D}(y) = F(x)$ transforms into $u_1'y_1' + u_2'y_2' = F(x)$.

Problem 6. Use the method of variation of parameters to determine the general solution of the differential equation $y''' - y'' + y' - y = e^{-x}\sin(x)$.

(a) (10 points) Solve the complementary equation $y''' - y'' + y' - y = 0$.

(b) (15 points) Compute the Wronskian $W(x)$ and determinants of auxiliary matrices, $W_1(x), W_2(x)$ and $W_3(x)$.

(c) (5 points) Use the formulas

$$u_1(x) = \int \frac{F(x)W_1(x)}{W(x)},\ u_2(x) = \int \frac{F(x)W_2(x)}{W(x)},\ u_3(x) = \int \frac{F(x)W_3(x)}{W(x)}$$

to present the general solution $y = u_1y_1 + u_2y_2 + u_3y_3$ in integral form.