MATH 146B: Ordinary and Partial Differential Equations

Bonus 1

Reviewing page 2 of Lecture 7 notes may provide valuable insights.

Problem 1. Find the Taylor series expansion and determine the radius of convergence for the following functions.

(a) (3 points) $\frac{1}{(a-x)^2}$.

Hint: notice that $\frac{1}{(a-x)^2} = \left(\frac{1}{a-x}\right)'$ and $\frac{1}{a-x} = \frac{1}{a}\sum_{n\geq 0}\left(\frac{x}{a}\right)^n$.

(b) (2 points) $\frac{1}{(a-x)^3}$.

(c) (2 points) $\frac{1}{(a-x)^k}$, $k > 3$.

(d) (3 points) Consider the rational function $f(x) = \frac{P(x)}{Q(x)}$, where $Q(x)$ has roots $x_1, x_2, \ldots, x_k$, possibly with nontrivial multiplicities. What does your conclusion in part (c) imply about the radius of convergence for the Taylor series of $f(x)$ centered at zero?