MATH 11: Introduction to Discrete Structures

Midterm Review

Solutions

Problem 1. Let p represent the statement "I have coffee" and q represent the statement "I am awake".

Translate the symbolic expressions into plain English.

(a) $\neg p$

Solution. "I do not have coffee."

(b) $p \wedge q$

Solution. "I have coffee and I am awake."

(c) $p \vee q$

Solution. "I have coffee or I am awake."

(d) $q \vee (\neg p)$

Solution. "I am awake or I do not have coffee."

Translate the following plain English statements into symbolic language using the symbols p, q, $\wedge$, $\vee$, and $\neg$.

(a) "I have coffee."

Solution. p

(b) "I have coffee and I am awake."

Solution. $p \wedge q$

(c) "Either I have coffee or I am awake."

Solution. $p \vee q$

(d) "I am not awake or I do not have coffee."

Solution. $\neg q \vee \neg p$

Problem 2. Construct the truth tables for the following compound statements.

(a) $\neg(p \vee q)$.

Solution.

p	q	$p \vee q$	$\neg(p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

(b) $\neg(p \vee q) \wedge r$.

Solution.

p	q	r	$p \vee q$	$\neg(p \vee q)$	$\neg(p \vee q) \wedge r$
T	T	T	T	F	F
T	T	F	T	F	F
T	F	T	T	F	F
T	F	F	T	F	F
F	T	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T
F	F	F	F	T	F

(c) $p \vee (q \wedge \neg r)$.

Solution.

p	q	r	$\neg r$	$q \wedge \neg r$	$p \vee (q \wedge \neg r)$
T	T	T	F	F	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	T	F	T
F	T	T	F	F	F
F	T	F	T	T	T
F	F	T	F	F	F
F	F	F	T	F	F

Problem 3. Use truth tables to show that the propositions $\neg(p \wedge \neg q)$ and $\neg p \vee q$ are logically equivalent.

Solution.

p	q	$\neg q$	$p \wedge \neg q$	$\neg(p \wedge \neg q)$
T	T	F	F	T
T	F	T	T	F
F	T	F	F	T
F	F	T	F	T

p	q	$\neg p$	$\neg p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

Since the truth values for both propositions are the same for all possible combinations of p and q, we conclude that $\neg(p \wedge \neg q)$ and $\neg p \vee q$ are logically equivalent.