Spring 2024

MATH 11: Introduction to Discrete Structures

Homework 3

Problem 1. (10 points) Consider the Ackermann function, a two-argument function where each argument takes a non-negative integer value. It is defined as follows:

(I) If $m = 0$, then $A(m, n) = n + 1$.

(II) If $m \neq 0$ but $n = 0$, then $A(m, n) = A(m - 1, 1)$.

(III) If $m \neq 0$ and $n \neq 0$, then $A(m, n) = A(m - 1, A(m, n - 1))$.

Compute the values below. Show steps similar to the example on page 3 of Lecture 7 notes. You may use the value $A(1, 1)$ computed there.

(a) (5 points) $A(1, 2)$.

(b) (5 points) $A(2, 1)$.

Problem 2. (10 points) Use Euclid's algorithm to find the greatest common divisor (GCD) of the following pairs of numbers. Show your steps similar to the example on page 1 of Lecture 8 notes.

(a) (5 points) $\mathrm{GCD}(132, 55)$.

(b) (5 points) $\mathrm{GCD}(357, 112)$.

Problem 3. (10 points) Suppose you have a list of three numbers: 1, 2, and 3. There are $3! = 6$ possible initial arrangements: $A_1 = [1, 2, 3]$, $A_2 = [1, 3, 2]$, $A_3 = [2, 1, 3]$, $A_4 = [2, 3, 1]$, $A_5 = [3, 1, 2]$, $A_6 = [3, 2, 1]$. We would like to use the bubble sort algorithm to rearrange (sort) the elements in increasing order.

(a) (5 points) For each of the six arrangements, find the number of swaps produced by the bubble sort algorithm.

(b) (5 points) Find the average number of swaps produced by the bubble sort algorithm by computing the arithmetic mean of the numbers you obtained in part (a).

Problem 4. (10 points) Using the definition provided at the bottom of page 2 in Lecture 8 notes, determine whether the following statements hold.

(a) (5 points) $5x + \ell n(x) = \mathcal{O}(x)$.

(b) (5 points) $e^{5x} + x^2 = \mathcal{O}(e^x)$.