MATH 11: Introduction to Discrete Structures

Homework 2

Problem 1. (20 points) Consider the set $A = \{a, b, c, d\}$, and relation

$$\mathcal{R} = \{(a, a), (a, b), (a, c), (b, d), (d, a), (b, c), (c, b), (b, b), (d, d)\}.$$

(a) (5 points) Draw the directed graph representing $\mathcal{R}$.

(b) (5 points) Find the reflexive closure of $\mathcal{R}$.

(c) (5 points) Find the symmetric closure of $\mathcal{R}$.

(d) (5 points) Find the transitive closure of $\mathcal{R}$.

Problem 2. (10 points) Recall that a relation is called *antisymmetric* if for all $x, y \in A$, if $(x, y) \in \mathcal{R}$ and $(y, x) \in \mathcal{R}$, then $x = y$.

Consider the set $A = \{a, b, c\}$, and the relation $\mathcal{R} = \{(a, a), (a, b), (a, c), (b, c), (c, b)\}$. Explain why $\mathcal{R}$ does not have an antisymmetric closure.

Problem 3. (25 points) Given the set $A = \{1, 2, 3\}$, define a relation $\mathcal{R} \subseteq A \times A$ that satisfies the required properties. You can either explicitly list the elements in $\mathcal{R}$ or draw the directed graph representing it.

(a) (5 points) $\mathcal{R}$ is symmetric and reflexive, but not transitive.

(b) (5 points) $\mathcal{R}$ is transitive and reflexive, but not symmetric.

(c) (5 points) $\mathcal{R}$ is transitive but not reflexive and not symmetric.

(d) (5 points) $\mathcal{R}$ is transitive, reflexive and symmetric.

(e) (5 points) $\mathcal{R}$ is not transitive, not reflexive and not symmetric.

Problem 4. (15 points) Consider the set $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ of residues modulo 10. Each relation on A below possesses the properties of being transitive, reflexive, and symmetric. Determine the corresponding partition of A into equivalence classes for each relation.

Example. The relation $\mathcal{R}$ is defined as follows: $a\mathcal{R}b$ if and only if $a \equiv b \pmod{10}$. This relation consists of pairs of elements $\{(0, 0), (1, 1), \dots, (9, 9)\}$, indicating that each element is equivalent only to itself. The corresponding partition of A is a disjoint union of ten one-element sets:

$$A = \{0\} \sqcup \{1\} \sqcup \{2\} \sqcup \{3\} \sqcup \{4\} \sqcup \{5\} \sqcup \{6\} \sqcup \{7\} \sqcup \{8\} \sqcup \{9\}.$$

(a) (5 points) $a\mathcal{R}b$ if and only if $a \equiv b \pmod{5}$.

(b) (5 points) $a\mathcal{R}b$ if and only if $a \equiv b \pmod{2}$.

(c) (5 points) $a\mathcal{R}b$ if and only if $a \equiv b \pmod{3}$.