MATH 11: Introduction to Discrete Structures

Homework 1

Problem 1. (10 points) Consider the universal set $U = \{1, 2, 3, \ldots, 9\}$ and sets $A = \{1, 2, 5\}$, $B = \{2, 3, 5, 7\}$, $C = \{1, 3, 5, 7, 9\}$. Determine the following sets.

(a) $A \cap B$ and $A \cap C$

(b) $A \cup (B \cap C^c)$

(c) A^c

(d) $A \setminus B$ and $A \setminus C$

(e) $A \oplus B$ and $(A \oplus B) \setminus C$

Problem 2. (20 points) Let C_n be the half-open interval $I_n = \left(\frac{2}{n+1}, \frac{2}{n}\right]$ for each positive integer n.

(a) (6 points) Sketch the first three intervals I_1, I_2 and I_3, find the union $I_1 \cup I_2 \cup I_3$ and intersection $I_1 \cap I_2 \cap I_3$.

(b) (7 points) Find the union $\bigcup_{n=1}^{\infty} I_n$.

(c) (7 points) Find the intersection $\bigcap_{n=1}^{\infty} I_n$.

Problem 3. (20 points) Consider the set $X = \{1, 2, 3, 4, 5\}$. For each collection of subsets in (a) − (c) below, determine if the collection is a partition of X. If the collection is a partition of X, put a check mark (✓) next to it. Otherwise, explain why it is not a valid partition.

(a) (5 points) $X_1 = \{1, 2\}, X_2 = \{2, 3, 4, 5\}$.

(b) (5 points) $X_1 = \{1, 2\}, X_2 = \{4, 5\}, X_3 = \{3\}$.

(c) (5 points) $X_1 = \{1\}, X_2 = \{2, 3, 4, 5\}, X_3 = \varnothing$.

(d) (5 points) List all partitions $X = X_1 \sqcup X_2$ with $|X_1| = 4$ and $|X_2| = 1$.

Problem 4. (15 points) At a store offering chocolate chip and oatmeal raisin cookies, 90 customers purchased chocolate chip cookies, 80 bought oatmeal raisin cookies, and 30 bought both types. Additionally, 15 customers did not purchase any cookies. How many customers visited the store in total?

Problem 5. (15 points) **Use mathematical induction** to prove that $6^n - 1$ is divisible by 5 for all positive integers n. **Hint:** $6^{k+1} = (5 + 1)6^k$.

Problem 6. (20 points) The Fibonacci sequence is defined recursively as follows:

$$F_1 = 1, \quad F_2 = 1$$

and for $n \geq 3$,

$$F_n = F_{n-1} + F_{n-2}.$$

Prove by induction that the n-th Fibonacci number F_n satisfies the inequality $F_n < 2^n$ for all positive integers n.