MATH 11: Introduction to Discrete Structures

Final Review

Problem 1. (2 points) If you roll two fair six-sided dice, what is the probability of getting

(a) a sum between 7 and 9 (inclusive)?

(b) a sum strictly less than 7 or strictly greater than 9?

Problem 2. (3 points) Suppose we have a random experiment with sample space S and two events A and B. Determine whether the following statements are true or false, and justify your answer:

(a) For any two events A and B, we have $P(A \cap B) = P(A)P(B)$.

(b) If events A and B are mutually exclusive, then $P(A \cap B) = 0$.

(c) If events A and B are independent, then $P(A|B) = P(A)$.

Problem 3. (3 points) Let X be a discrete random variable with the following probability mass function:

$$P(X = k) = \begin{cases} 0.2 & \text{if } k = 1 \\ 0.3 & \text{if } k = 2 \\ 0.5 & \text{if } k = 3 \\ 0 & \text{otherwise} \end{cases}$$

Calculate the following probabilities.

(a) $P(X = 2)$

(b) $P(X < 3)$

(c) $P(X \geq 2)$

Problem 4. (1 point) Consider two events A and B such that $P(A) = 0.3$ and $P(B) = 0.4$. If A and B are independent, what is $P(A \cap B)$?

Problem 5. (1 point) Consider two events E and F such that $P(E) = 0.6$ and $P(F) = 0.7$. If E and F are independent, what is $P(E \cup F)$?

Problem 6. (1 point) Let X be a random variable representing the number of heads obtained when flipping a coin 3 times. Find the probability mass function of X.

Problem 7. (2 points) Let $\mathcal{S}$ be a random variable representing the time it takes for a computer program to execute. The mean execution time is 100 seconds, and the variance is 25 seconds.

(a) Use Markov's inequality to estimate the probability that the program's execution time is at least 5 minutes.

(b) Use Chebyshev's inequality to give the lower bound on the probability that the program's execution time is within 15 seconds of the mean.