# Lecture 26

## Transformation of trigonometric functions

MATH 0200

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# Outline

# Amplitude

**Definition**

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**Example**

Let's find the amplitude of the function $f(x) = 5\cos(x)$. Recall that the range of cosine is the closed interval $[-1, 1]$, hence, the range of $f(x)$ is $[-5, 5]$ and the amplitude equals $(5 - (-5))/2 = 5$.

## Remark

A shift of a function $f(x)$ will likely change the maximal and minimal values, but will **NOT** change the difference between them. Therefore shifts preserve amplitude.

## Example

Let's find the amplitude of the function
$f(x) = 5\cos(x-2)+7$.

## Remark

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## Example

Let's find the amplitude of the function
$f(x) = 5\cos(x-2)+7$.
The range of cosine $\cos(x-2)$ is still the closed interval $[-1,1]$, hence, the range of $f(x)$ is $[2,12]$ (here $2 = 5\cdot(-1)+7$ and $12 = 5\cdot 1+7$), so the amplitude equals $(12-2)/2 = 5$ (as before).

# Period



**Definition**

Let $f$ be a function and $p$ a positive number. We say that $p$ is the **period** of $f$ if $p$ is the smallest positive number with $f(x+p) = f(x)$ for every real number $x$ in the domain of $f$.

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## Remark

Some functions do not repeat their behavior at regular intervals and thus do not have a period. For instance, any linear function $f(x) = mx + b$ (with $m \neq 0$) does not have a period. A function is called **periodic** if it has a period.

## Example

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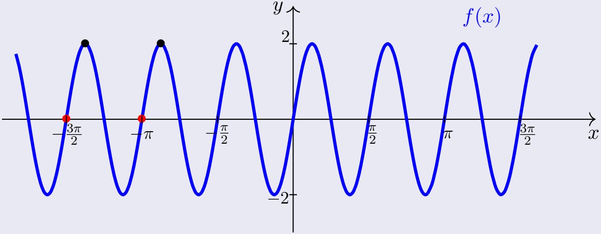

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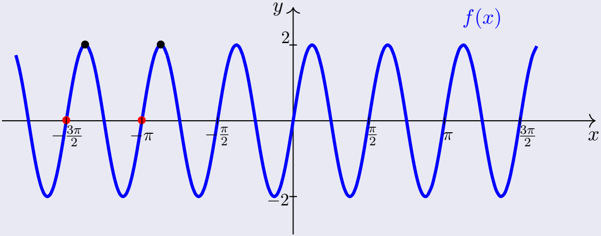


2. The amplitude of $f(x)$ is equal to $\frac{2-(-2)}{2} = 2$.

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**Remark**

- Shifts and vertical stretch have no effect on the period.
- The period of $\sin(mx)$ and $\cos(mx)$ is $p = \frac{2\pi}{m}$.

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**Question**

Find the amplitude and period of $f(x) = 3\sin(7x) - 2$.

**Answer:** the range of $f(x)$ is the closed interval $[-5, 1]$, so the amplitude equals $\frac{1-(-5)}{2} = 3$. The period is $p = \frac{2\pi}{7}$.