Lecture 25

More trigonometric formulas

MATH 0200

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Outline

Double-angle formulas

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2. Let's pretend that we don't know the value of $\tan(60°)$, but know that $\tan(30°) = \frac{1}{\sqrt{3}}$.
   We get $\tan(60°) = \frac{2\tan(30°)}{1-\tan^2(30°)} = \frac{\frac{2}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{2}{\sqrt{3}\left(1-\frac{1}{3}\right)} = \frac{2}{\frac{2\sqrt{3}}{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$.

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2. Find $\cos(75°)$.
   Notice that $75 = 150/2$ and $\cos(150°) = -\cos(30°) = -\frac{\sqrt{3}}{2}$. We get
   $\cos(75°) = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2}$.

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   We notice that $105 = 60 + 45$, therefore
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2. Express $\cos\left(a + \frac{\pi}{3}\right)$ in terms of $\sin(a)$ and $\cos(a)$.
   $\cos\left(a + \frac{\pi}{3}\right) = \cos(a)\cos\left(\frac{\pi}{3}\right) - \sin(a)\sin\left(\frac{\pi}{3}\right) = \frac{1}{2}\cos(a) - \frac{\sqrt{3}}{2}\sin(a)$.