Lecture 21

Inverse trigonometric functions

MATH 0200

Dr. Boris Tsvelikhovskiy

Outline

1. The inverses of $\cos(x), \sin(x)$ and $\tan(x)$

2. Definitions of arccosine, arcsine and arctangent

Arccosine, the inverse of $\cos(x)$

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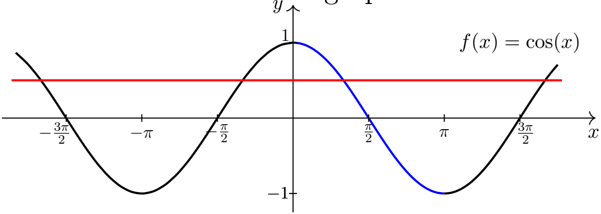

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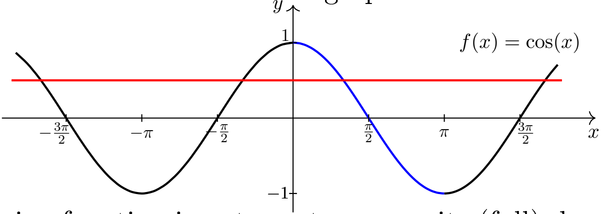

The cosine function is not one-to-one on its (full) domain $(-\infty, \infty)$. However, it is one-to-one if we restrict the domain, for instance, to $[0, \pi]$ and, therefore is invertible on the interval $[0, \pi]$:

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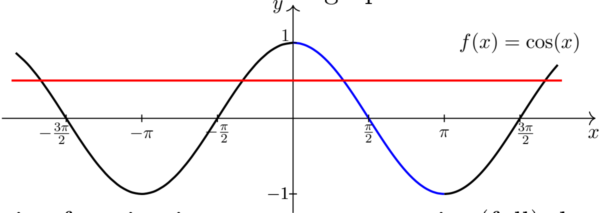

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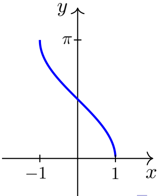
$f(x) = \arccos(x)$
Domain: $[-1, 1]$
Range: $[0, \pi]$



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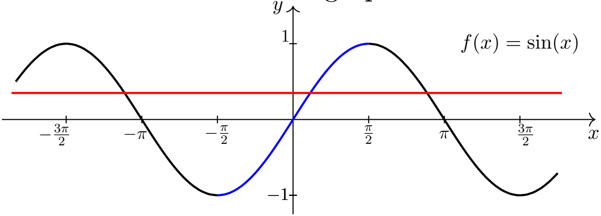

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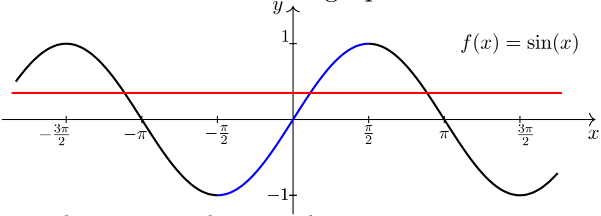

Similar to the cosine, the sine function is not one-to-one on its (full) domain $(-\infty, \infty)$. However, it is one-to-one if we restrict the domain, for instance, to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and, therefore is invertible on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$:

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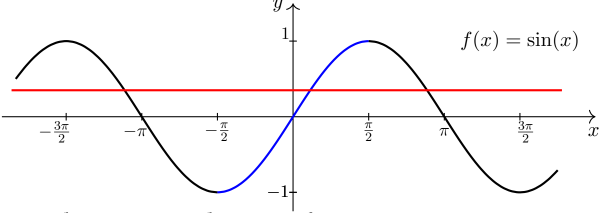

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$f(x) = \arcsin(x)$

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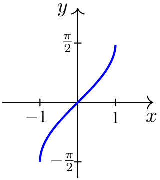
Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



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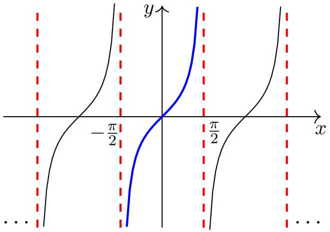

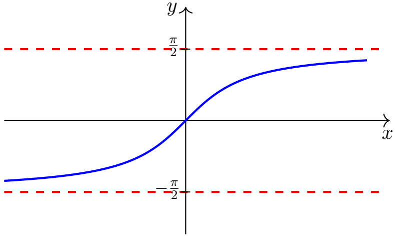

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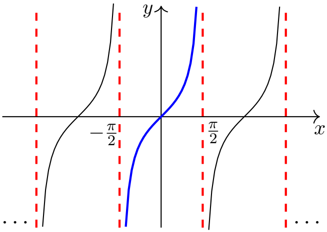

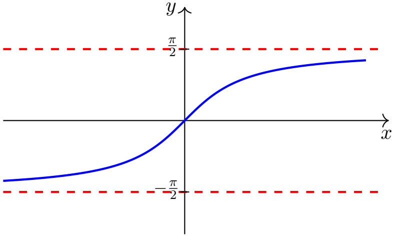

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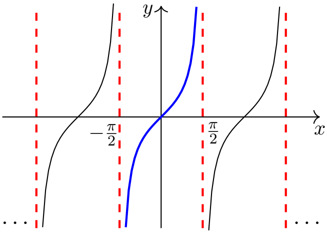

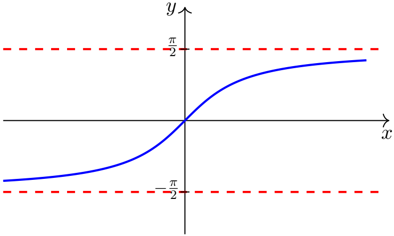

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- $\arcsin(0.5) = \frac{\pi}{6}$ as $0 \le \frac{\pi}{6} \le \pi$ and $\sin\left(\frac{\pi}{6}\right) = 0.5$;

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- $\arctan(\sqrt{3}) = \frac{\pi}{3}$ as $-\frac{\pi}{2} < \frac{\pi}{3} < \frac{\pi}{2}$ and $\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$;

Question

Evaluate $\arctan(\tan(-5))$ (round your answer to **three** decimal places).

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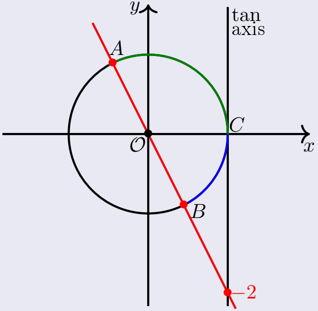

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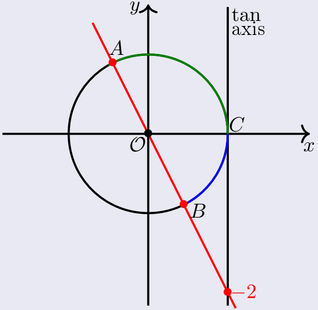

Notice that $\arctan(-2)$ is a negative number, minus the length of the arc CB, shaded in blue. The smallest positive value of x with $\tan(x) = -2$ is the length of the arc CA, shaded in green. It is equal to $\pi + \arctan(-2)$.

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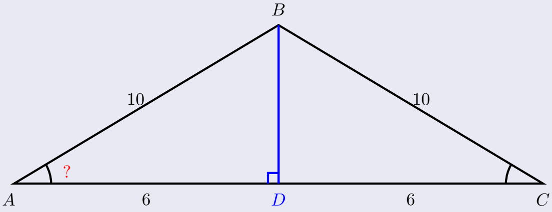

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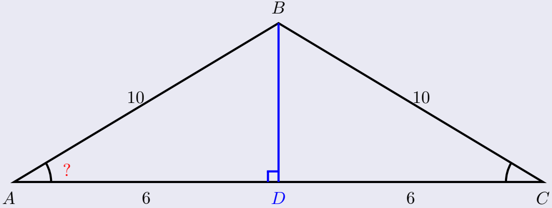

We drop a perpendicular from vertex B to the base AC and denote the point of intersection by D. Recall that since triangle $\triangle ABC$ is isosceles, D is the midpoint of AC, so $|AD| = |DC| = 12/2 = 6$. We get $\cos(\angle BAC) = \frac{6}{10} = 0.6$, hence, $\angle BAC = \arccos(0.6) \approx 53.13°$.