Lecture 1

Inequalities, sets and absolute value

MATH 0200

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Outline

Inequalities

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- If $a > b > 0$, then $\frac{1}{b} > \frac{1}{a}$.

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- $X = \{$  ,  $\}$ is a set which consists of two elements, a panda  and a dog .

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Let $A = (-1, 4)$ and $B = [-5, 2]$ be two intervals. Then the union $A \cup B$ is the half-open interval $[-5, 4)$ and the intersection $A \cap B$ is the half-open interval $(-1, 2]$.

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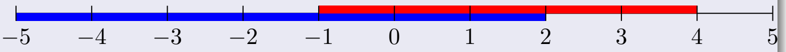

Question

Let $X = \{1, 4, 5, 6, 8, 9, 11\}$ and $Y = \{2, 4, 7, 9\}$ be two sets. What are the union $X \cup Y$ and intersection $X \cap Y$?

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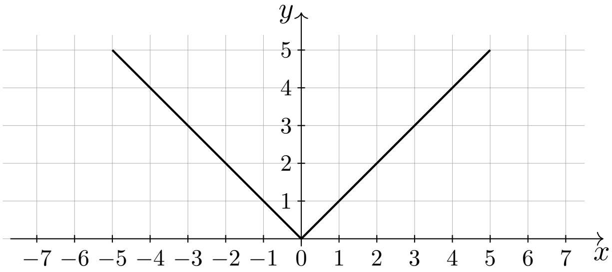

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