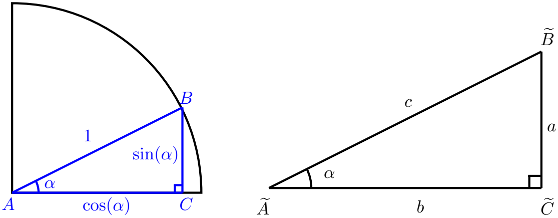
Lecture 19

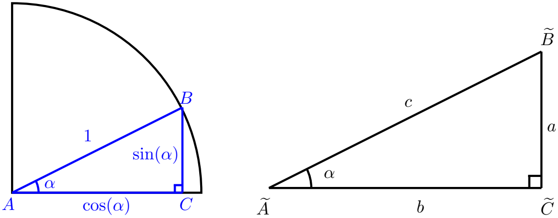
Trigonometry in right triangles

MATH 0200

Dr. Boris Tsvelikhovskiy

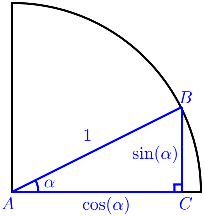
Outline

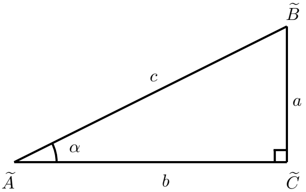

B
1
sin(α)
α
A
cos(α)
C
B̃
c
a
α
Ã
b
C̃



Remark

We observe that our first trigonometric identity $\sin^2(\alpha) + \cos^2(\alpha) = 1$ is nothing else but Pythagorean theorem for triangle $\triangle ABC$.





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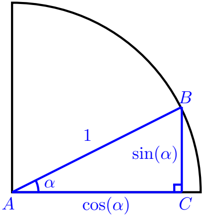

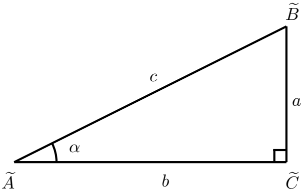

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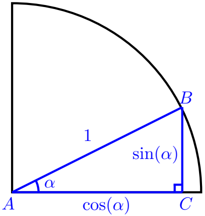

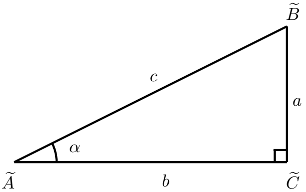

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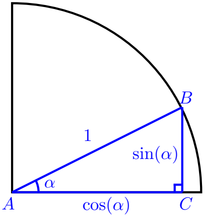

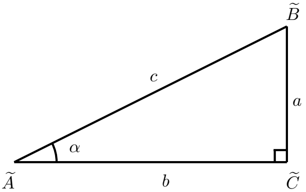

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Consider the right triangle $\triangle ABC$ below with legs $AC = 12$ and $BC = 5$.

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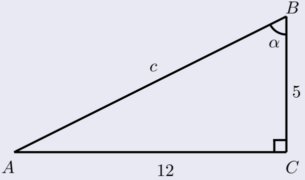

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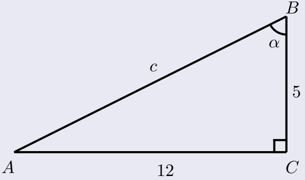

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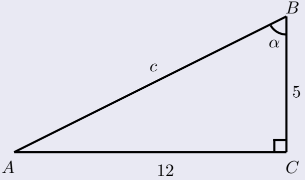

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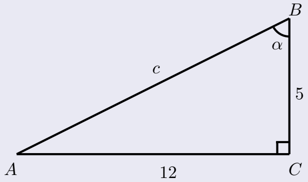

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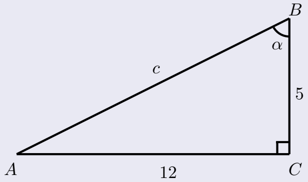

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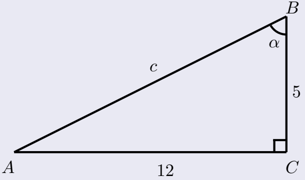

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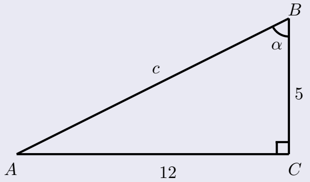

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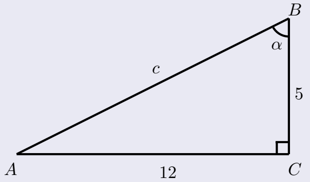

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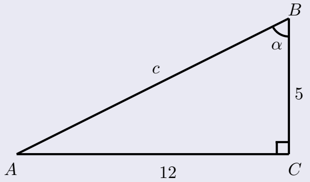

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(c) Evaluate $\cos(\alpha)$. We get $\cos(\alpha) = \frac{5}{13}$.

(d) Evaluate $\cot(\alpha)$. We get $\cot(\alpha) = \frac{\cos(\alpha)}{\sin(\alpha)} = \frac{5}{13} \cdot \frac{13}{12} = \frac{5}{12}$.

Example

Suppose a 20-foot ladder is leaning against a wall, making a 72° angle with the ground. How high up the wall is the end of the ladder?

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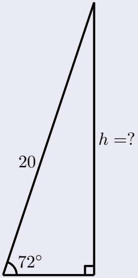
Suppose a 20-foot ladder is leaning against a wall, making a $72°$ angle with the ground. How high up the wall is the end of the ladder?



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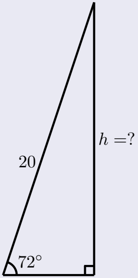
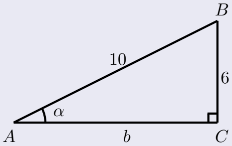


Solution: $\sin(72°) = \frac{h}{20} \Leftrightarrow h = 20\sin(72°) \approx 19.021$ ft.

Consider the right triangle depicted below.

Consider the right triangle depicted below.

Question



- Evaluate $\sin(\alpha)$.

Consider the right triangle depicted below.

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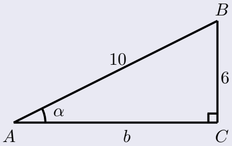

- Evaluate $\sin(\alpha)$.
- Evaluate $\tan(\alpha)$.