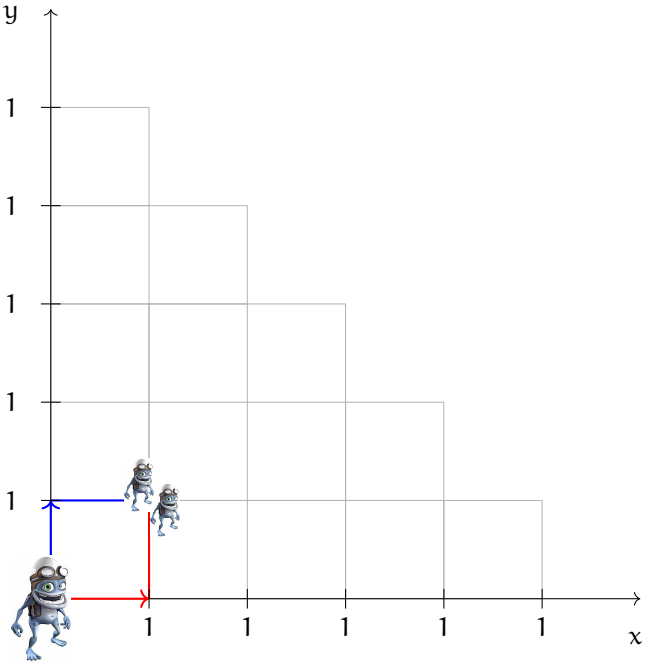
MATH 0200: Preparation for Scientific Calculus

Froginomial Bonus

Problem. Crazy Frog[1] jumps on the grid. He starts at the origin $(0,0)$ and each time hops either one step right or one step up.

(a) [2 **pts**] Find the number of paths (ways) the frog can hop to each point on the grid below (write the number you get next to the vertex). For instance, there are two paths to $(1,1)$: either first hop 'right' and then 'up' or first hop 'up' followed by 'right' (the 'red' and 'blue' paths below).



[1] Crazy Frog, originally known as The Annoying Thing, is a Swedish CGI-animated character and musician created in 2003 by actor and playwright Erik Wernquist.

(b) Now substitute every path with a monomial, writing a for a 'right' hop and b for an 'up' hop. For instance, we have 4 'two-hop' paths:

$$\begin{aligned} \rightarrow\rightarrow \quad &\longleftrightarrow \quad aa \rightarrow a^2 \\ \rightarrow\uparrow \quad &\longleftrightarrow \quad ab \rightarrow ab \\ \uparrow\rightarrow \quad &\longleftrightarrow \quad ba \rightarrow ab \\ \uparrow\uparrow \quad &\longleftrightarrow \quad bb \rightarrow b^2. \end{aligned}$$

This implies $(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2$. The choice of the direction of the hop is the same as the choice of a or b in the corresponding $(a+b)$ factor. Using your answer in (a), expand

(1) [1 **pts**] $(a+b)^3$ (use '3-hop' paths).

(2) [1 **pts**] $(a+b)^4$ (use '4-hop' paths).

(3) [1 **pts**] $(a+b)^5$ (use '5-hop' paths).